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Short communication

Nonlinear predictive control of a molten carbonate fuel cell stack

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ABSTRACT

Operating temperature of a molten carbonate fuel cell stack should be controlled within a special range in order to improve the performance of fuel cell. In this paper, a nonlinear predictive control algorithm based on the Takagi–Sugeno fuzzy model is developed for the temperature of a molten carbonate fuel cell stack. Through predicting the outputs on a Takagi–Sugeno fuzzy model, a discrete optimization of the control action is adopted according to the principle of branch-and-bound method. The simulation results show the potential to introduce the predictive control based on Takagi–Sugeno fuzzy model for the development of fuel cells.

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1. Introduction

The molten carbonate fuel cell (MCFC) is one of the fuel cell technologies that have proven efficiency and environmental performance [1]. The performance and reliability of MCFC depend on its operating temperature greatly. The temperature for stable operating of MCFC ranges from 873 K to 973 K, and the nominal operating temperature is around 923 K. When the operating temperature is below 873 K, the activity of molten salts degrades and the performance of cells drops significantly. A higher operating temperature can improve the working voltage and output performance of MCFC. However, when the operating temperature is above 973 K, material corrosion accelerates greatly, and electrolyte loss increases, which increases the risk of short-circuit and shortens the stack lifespan [2]. Thus, controlling the operating temperature within a specified range and reducing temperature fluctuation are highly desirable.

Over the last decades, most of the researches in fuel cell control field were model-based [3,4]. Some researchers have made great progress on MCFC modeling to improve its performance. Most of the existing mathematical models have been established on the internal mechanisms, ranging from a one dimensional non-isothermal model to a three dimensional non-isothermal and non-isobaric models [5–7], but it is too difficult to follow these models to design a control system, especially in the design of the online control. To meet the demands of developing valid control strategies, the data-

driven approach has been developed to establish novel fuel cell models [8–11]. In Ref. [11], a nonlinear Takagi–Sugeno (T–S) fuzzy model of a MCFC stack is built with an identification method, and identified fuzzy model can efficiently approximate the static and dynamic behavior of a MCFC stack. The T–S fuzzy model can be used to predict the variants responses on-line and make it possible to design online controller of a MCFC stack.

This paper focuses on the application of predictive control based on T-S fuzzy model to MCFC stack temperature. By the optimization approach and the explicit use of a process model, model predictive control can handle multivariable processes with nonlinearities, non-minimum phase behavior, and can efficiently deal with constraints. Model predictive control has been one of the most attractive control techniques in the chemical and petrochemical industries during the past decades. A model predictive controller has been developed in ref. [12] for a hybrid PEMFC system with ultracapacitors as an auxiliary source of power. Like any other model-based control, model predictive control relies greatly on process models. An accurate process model is required if the process is to be regulated tightly. In our case, the T-S fuzzy model of MCFC stack is obtained by the identification method from ref. [11] is the nonlinear predictive model. In the presence of nonlinearities and constraints, usually a nonconvex optimization problem must be solved in each sampling period in order to obtain the future control action. Different algorithms such as sequential quadratic programming (SQP) or branch-and-bound, can be used to avoid this problem. However, SQP usually converges to local minimum, giving poor solutions. In this paper, optimal control action in the discrete space is





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searched based on the principle of the branch-and-bound method [13].

This paper is organized as follows. Section 2 mainly introduces the structure of fuzzy predictive control system of MCFC. Section 3 describes the optimization of control action. In Section 4, MCFC control simulations are provided, followed by conclusions in Section 5.

2. Structure of fuzzy predictive control system of MCFC

Fig. 1 shows the scheme of fuzzy predictive control system of MCFC, which consists of a controlled plant, a predictive model and a controller implemented by an optimizer. The MCFC dynamic physical model described in ref. [14] is built to mimic the real controlled plant of a 10 kW MCFC stack. The stack temperature T° is controlled variable. Flow rates of anode and cathode (F_a , F_c) are chosen as manipulated variables. The current density J is considered to be a disturbance.

In Fig. 1, y(k) is the current value of stack temperature; y_{ref} is the reference curve of stack temperature; $\hat{y}(k + 1)$ is the predicted next value; U(k) is the manipulated variable. The T–S fuzzy model predicts future stack temperature values using history stack temperature values. Based on the difference between the next reference values and the predicted stack temperature values, the optimal controller can determine the next control signal for MCFC.

In this paper, we use the following form to facilitate modeling and control designing:

$$T^{\circ}(k+1) = f(F_{a}(k), F_{c}(k), J(k), T^{\circ}(k))$$
(1)

where *f* denotes the nonlinear relation. As a model of the stack to be identified to simulate dynamically the variation of temperature under various flow rates and disturbance.

Let u ($u \in U \subseteq \mathbb{R}^r$) be input variable and y be output variable. We define:

$$\{u_j(k)\}_0^{\mathrm{nuj}} = [u_j(k), \ldots, u_j(k - \mathrm{nuj} + 1)] \cdot (j = 1, \ldots r,)$$

 $\{y(k)\}_{0}^{ny} = [y(k), \dots, y(k - ny + 1)]$ with nuj and ny being the order of u_j and y, respectively, then multiple inputs and single output (MISO) system can be denoted as follows,

$$y(k+1) = f(x(k))$$
 (2)

with $x(k) = [\{u_1(k)\}_0^{nu1}, \dots, \{u_r(k)\}_0^{nur}, \{y(k)\}_0^{ny}] = [x_{k1}, \dots, x_{kn}]$ is the regression data vector consisting of input/output data at the *k*th instant and before.

The T–S predictive model with linear consequents employed to fit the MISO system in this paper is a collection of fuzzy rules, which



Fig. 1. T-S fuzzy model-based predictive control strategy for MCFC.

is in the form of "*If*... *then*...". The *i*th rule of the output $\hat{y}_i(k+1)$ is given by

$$R_i$$
: If $x(k)$ is A_i , then

$$\hat{y}_i(k+1) = p_{i,0} + p_{i,1}x_{k1} + \dots + p_{i,n}x_{kn} i = 1, \dots c$$
(3)

where *c* is the number of rules, $A_i = \{A_{i,1} \dots A_{i,n}\}$ is the set of membership functions associated to the *i*th rule and $p_i = [p_{i,0}, p_{i,1}, \dots, p_{i,n}]$ is the parameter vector of the *i*th submodel (rule).

The T–S fuzzy model can be obtained by the antecedent and consequent identification. Antecedent identification is implemented by fuzzy clustering based on the principle of Fuzzy C-Means (FCM) algorithm. The consequent part of the fuzzy rule is identified by using the Kalman filtering algorithm. The details of identification can be found in ref. [11].

3. Optimization of control action

In model predictive control, a process dynamic model is used to predict future outputs over a prescribed period [15]. A sequence of future control actions is computed using this model by optimizing objective function. Based on T–S fuzzy model, the future process outputs $\hat{y}(k + i)$ for i = 1, ..., P, are predicted over the prediction horizon *P*. These values depend on the current process state, and on the future control signals u(k + i) for i = 1, ..., M, where $M \le P$ is the control horizon. The control variable is manipulated only within the control horizon and remains constant afterwards, u(k + i) = u(k + M - 1)for i = M, ..., P - 1. The optimal control action sequences (j = 1, ...r), will be solved to minimize the objective function

$$J = \sum_{i}^{P} q_{i} [y_{\text{ref}}(k+i) - \hat{y}(k+i)]^{2} + \sum_{j=1}^{r} \sum_{i=1}^{M} \lambda_{ji} [u_{j}(k+i) - u_{j}(k+i-1)]^{2}$$
(4)

The first term accounts for minimizing the variance between the process output and the reference, while the second term represents a penalty on the control effort. Eq. (4) is used in combination with input and output constraints,

$$u_{j\min} \le u_j \le u_{j\max}, y_{\min} \le y \le y_{\max}$$
(5)

To implement the receding horizon optimization control, only the first control action in the sequence is applied, the horizons are moved one sample period towards the future, and optimization is repeated. In this optimization problem, control sequences are optimized according to the principle of branch-and-bound method which is a tree structure search technique and requires a discretization of the control space. The process of optimization consists of determining the discrete space of control action and searching the optimal control sequence [13].

3.1. Determine the discrete space

In this subsection, the tree structure space of control sequence is determined.

First, the error e(k) between the reference value and the actual output of the MCFC system at the current sampling period, and the error change rate ec(k) should be calculated,

$$e(k) = y_{\text{ref}}(k) - y(k) \tag{6}$$

$$ec(k) = e(k) - e(k-1)$$
 (7)

Second, *e* and ec are discretized into *E* and EC in their respective fuzzy discourse domain, then the increment $du_i(k)$ of $u_i(k)$ (j = 1, ..., r)



Fig. 2. The tree structure discrete space of all the control variables.

is inferred according to the following analytical fuzzy reasoning formula,

$$du_i(k) = \beta[\alpha_i E + (1 - \alpha_i)EC]$$
(8)

where β and α_j are the undetermined coefficients. The consecutive space of $u_i(k)$, centered on $u_i(k-1)$, is given by:

$$B = (u_j(k-1) - \operatorname{abs}(\operatorname{d} u_j(k), u_j(k-1) + \operatorname{abs}(\operatorname{d} u_j(k)))$$
(9)

Third, the consecutive space of $u_j(k)$ is discredized into the discrete one. In the same way, based on each discrete search point of $u_j(k+i-1)$, discrete search space of $u_j(k+i)$ can be obtained in the control horizon $(1 \le i \le M - 1)$. The control sequence does not branch anymore during the sampling period beyond the control horizon $(M \le i \le P - 1)$.

As a result, the discrete space of control sequence can be obtained. Fig. 2 illustrates the tree structure discrete space of all the control variables with predictive horizon *P* and control horizon *M*. In this figure, $u_j^t(k + i)$ ($t = 1, ..., n_j^{k+i}$) denotes the *t*th discrete search point in discrete space of u_j at the k + ith sample instant. This part of the algorithm embodies the branch idea in the branch-and-bound method.

3.2. Search optimal control sequences

In the tree structure discrete space of control sequences, the optimal control sequences which minimize the objective function should be searched. At first, the greedy algorithm is employed to find a 'good' solution in order to set the initial upper boundary for the objective function (Eq. (4)). Given the selected $u_j(k+i-1)$ (j = 1, ..., r), the performance index as follows is computed for all discrete control actions $u_j^t(k+i)$ ($t = 1, ..., n_j^{k+i}$) (j = 1, ..., r) of the k+ith (i = 0, ..., P-1) level in Fig. 2:

$$J_{t}(k+i) = q_{i}[y_{\text{ref}}(k+i) - \hat{y}_{t}(k+i)] + \sum_{j=1}^{r} \lambda_{ji}[(u_{j}^{t}(k+i) - u_{j}(k+i-1))]^{2}$$
(10)

Then $J(k+i) = \min_{i} \{J_t(k+i)\}\)$ and corresponding $u_j(k+i)$ are found. $J_{\text{upper}} = \sum_{i=0}^{P-1} J(k+i)$ is the initial upper boundary of the objective function, and the control sequence composed of $u_j(k+i)$ $(j=1,\ldots,r, i=0,\ldots,P-1)$ is the 'good' solution mentioned above. Afterwards, the optimal control sequence should be searched according to the method used in ref. [16]. The first element $u_j(k)$ $(j=1,\ldots,r)$ of the optimal sequence is applied to the plant, and the obtained sample output y(k+i)is used to construct a new input/output data vector, x(k+1) =

Table 1

Parameters of the MCFC stack

Parameter	Unit	Value
Number of cells		25
Cell active area	m ²	0.4
Channel height of anode	m	$1.2 imes 10^{-3}$
Channel height of cathode	m	2×10^{-3}
Density of separator	kg m ⁻³	7900
Inlet temperature of anode	K	873
Inlet temperature of cathode	К	823
Inlet H ₂ of anode	Mole fraction	0.64
Inlet CO ₂ of anode	Mole fraction	0.2
Inlet H ₂ O of anode	Mole fraction	0.16
Inlet CO ₂ of cathode	Mole fraction	0.3
Inlet N ₂ of cathode	Mole fraction	0.553
Inlet Ω_2 of cathode	Mole fraction	0 147

 $[\{u_1(k+1)\}_1^{nu1}, \dots, \{y(k+1)\}_0^{ny}]$ which will be used in the next sample period.

4. Simulation

In this section, we present numerical simulations to illustrate the validation of the proposed predictive control developed for the temperature of MCFC stack based on T-S fuzzy model. As shown in Section 3, a T-S fuzzy model is set up at first, and the model predictive control is designed based on the T-S fuzzy model. For the identification of a MCFC T-S fuzzy model, the MCFC dynamic physical model described in ref. [14] is used to obtain the input/output data if it were the "true" MCFC stack. The parameters of a 10 kW MCFC stack are used in the simulation, and are given in Table 1. In order to obtain available identification data, the input signals of the MCFC dynamic physical model were uniformly random, including the anode flow rate and the cathode flow rate. To obtain values at integer time points, the fourth-order Runge-Kutta method was used to find the numerical solution to the MCFC dynamic physical model in the simulation. The input/output data was collected from the simulation, and then the fuzzy modeling algorithm in ref. [11] was employed to identify the T–S fuzzy model. To validate the T-S fuzzy model, it was used to perform dynamic simulation of the MCFC stack. Under various gas flow rates ($J = 1300 \text{ Am}^{-2}$), comparing the temperature values calculated by the T–S fuzzy model with the temperature data obtained from the simulation of the MCFC dynamic physical model, we obtained the results as in Fig. 3. From



Fig. 3. Identification results under variable flow rates.



Fig. 4. Responses of two control methods.



Fig. 5. Anti-disturbance result of the predictive control.

Fig. 3, we can see the obtained T–S fuzzy model can approximate the dynamic behavior of the MCFC dynamic physical model with good accuracy.

Next demonstration is the predictive control with the fuzzy model. The stack temperature is required to be kept constant (in general 923 K). The control simulations are performed for all the schemes with the following tuning parameters of predictive controller, prediction horizon P=15; control horizon M=5; controlled variables weights $q_i = 1.56$; manipulated variables weights $\lambda_{1,i} = 0.15$, $\lambda_{2,i} = 0.1$. A nonlinear PID controller is also used in the simulation. We use an improved nonlinear PID controller to control the stack. In the simulation, the parameters of PID are chosen as (see

[17] for details) $K_c = 0.8K_m$, $K_i = 0.3T_m$, $K_d = 0.1T_m$ where K_m and T_m are the proportion coefficient and the oscillation cycle of the step response of the system at the critical stability points applied with a proportional control. To adapt nonlinear dynamic, PID parameters above are tuned online as in [18]. We get the tracking curve of the controlled system shown in Fig. 4. The variant curve of the operating temperature when not controlled is labeled 'without control' in Fig. 4 and is stable at about 927 K. From Fig. 4, we can see the MCFC stack is controlled by the nonlinear predictive control algorithm based on T–S fuzzy model, which adjusts the operating temperature fluctuation and obtains satisfactory control effectiveness.

The anti-disturbance result is given in Fig. 5. And the simulation result is obtained at the condition with the current density changing (at time 8000 s, the current density stepping from 1300 Am^{-2} to 1100 Am^{-2}). The predictive controller is used to adjust the stack temperature to its steady value (923 K). From the simulation result we can find that the predictive control based on T–S fuzzy model is disturbance robust.

5. Conclusions

The operating temperature of the stack is an important controlled variable in the MCFC system. However, the existing mechanism model is too complicated to meet the design requirements of the control system. In this paper, a nonlinear predictive control algorithm based on the T–S fuzzy model is proposed. The validity of fuzzy predictive model of MCFC stack and the good performance of the nonlinear predictive controller are illustrated by simulations.

It is concluded that it is feasible to set up the model of the complex nonlinear MCFC stack system based on T–S fuzzy model and it can be used to predict the temperature responses online. The nonlinear predictive controller designed is efficient. It can control the stack temperatures to change smoothly to the ideal stabilization value, and shows robustness.

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